PHY180 Unit 11

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Abstract

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Formulas So Far:

Translational motion (constant acceleration)		Rotational motion (constant rotational acceleration)	
coordinate	x	rotational coordinate	ϑ
x component of displacement	$\Delta x = x_f - x_i$	change in rotational coordinate	$\Delta\vartheta = \vartheta_f - \vartheta_i$
x component of velocity	$v_x = \frac{dx}{dt}$	rotational velocity	$\omega_\vartheta = \frac{d\vartheta}{dt}$
x component of acceleration	$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$	rotational acceleration	$\alpha_\vartheta = \frac{d\omega_\vartheta}{dt} = \frac{d^2\vartheta}{dt^2}$
kinematics relationships (constant a_x):		rotational kinematics relationships (constant α_ϑ):	
	$v_{x,f} = v_{x,i} + a_x\Delta t$		$\omega_{\vartheta,f} = \omega_{\vartheta,i} + \alpha_\vartheta\Delta t$
	$x_f = x_i + v_{x,i}\Delta t + \frac{1}{2}a_x(\Delta t)^2$		$\vartheta_f = \vartheta_i + \omega_{\vartheta,i}\Delta t + \frac{1}{2}\alpha_\vartheta(\Delta t)^2$
		radial acceleration	$a_r = -\frac{v^2}{r} = -r\omega^2$
		tangential acceleration	$a_t = 0$

Some derivation:

$$\theta_f(t) = \theta_i + w_0 t + \frac{1}{2}\alpha_0 t^2$$

$$\overrightarrow{r}(t) = r\begin{bmatrix} cos\theta(t) \\ sin\theta(t) \end{bmatrix}$$

$$\overrightarrow{v}t = r \cdot w(t)\begin{bmatrix} -sin\theta(t) \\ cos\theta(t) \end{bmatrix}$$

$$w(t) = w_0 + \alpha_0 t$$

$$\overrightarrow{a}(t) = r\alpha_0 \begin{bmatrix} -sin\theta(t) \\ cos\theta(t) \end{bmatrix}$$

$$a_r = -w^2 r = -v^2/r$$

w is angular velocity (?)

Acceleration is towards center of rotation

$$\overrightarrow{a_t} \cdot \overrightarrow{a_r}$$

2 Cycling in a Velodrome

$$\hat{x} : N sin\theta = \frac{mv^2}{r}$$

$$\hat{y} : N cos\theta = mg$$

Angle of bank based on radius, velocity, and the constant g

$$\rightarrow tan\theta = \frac{v^2}{gr}$$

Rearranging:

$$v = \sqrt{g \cdot r \cdot tan(\theta)}$$

3 Rotational Inertia

Objects are harder to rotate the further you hold them from their center of mass.

More energy/momentum can be stored in an object revolving around a pivot further from te center of mass.

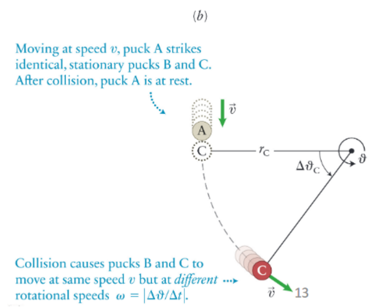
Kinetic energy in rotation

- Once energy transferred to "C"

$$K = \tfrac{1}{2} m\upsilon^2 = \tfrac{1}{2} m(r\omega)^2 = \tfrac{1}{2}(mr^2)\omega^2$$

- Can write the last term as

$$K_{\text{rot}} = \tfrac{1}{2} I\omega^2 \quad \text{with} \quad I \equiv mr^2,$$



- This is the rotational energy of puck C
  - Units are [J] just as we've always had
  - But I has units of [kg m^2]
  - Where m has units of [kg]

4 Kinetic Energy of Rigid Body

Choose axis of rotation: $x = y = 0$
Describe the rotational motion (angular frequency):

$$w_0 = \frac{d\theta a}{dt}$$

Consider one "piece" of object, a small slice of mass which is some distance $\overrightarrow{r}$ from the center of mass. All these small pieces will be moving with the same ω_r

$$\omega_\theta = \frac{2\pi}{T}$$

$$V = |\overrightarrow{r}|\,\omega_\theta$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m_n(r_n\omega_\theta)^2$$

All pieces of extended object sum:

$$K = \sum_{i=1}^{n} K_n$$

$$K_n = \frac{1}{2}m_n v_n^2$$

$$= \frac{1}{2} I_n \omega_\theta^2$$

$$\frac{1}{2} \sum_{i=1}^{n} I_n \omega_\theta^2$$

$$I = \sum_{i=1}^{n} I_n = \sum_{i=1}^{n} m_n r_n^2$$

Larger values of $I\omega$ it's easier to set an object int orotation.

$$I = mr^2$$